YENSEN

Starting Currents of Transformers

Electrical Engineering

E. E.

1913

STARTING CURRENTS OF TRANSFORMERS

BY

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M. S. University of Illinois, 1912

THESIS

Submitted in Partial Fulfillment of the Requirements for the

Degree of

ELECTRICAL ENGINEER

IN

THE GRADUATE SCHOOL

OF THE

UNIVERSITY OF ILLINOIS

1913

UNIVERSITY OF ILLINOIS

THE GRADUATE SCHOOL

April 10 1903

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

Trygve D. Yensen

ENTITLED Starting current of transformers

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Electrical Engineer

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Committee on Final Examination

UNIVERSITY OF ILLINOIS

ENGINEERING EXPERIMENT STATION

BULLETIN NO. 55 FEBRUARY, 1912

STARTING CURRENTS OF TRANSFORMERS

WITH SPECIAL REFERENCE TO TRANSFORMERS WITH SILICON STEEL CORES

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STARTING CURRENTS OF TRANSFORMERS

WITH SPECIAL REFERENCE TO TRANSFORMERS WITH SILICON STEEL CORES

I. Introduction

1. *Preliminary.*—It is generally known that, in closing the primary circuit of a transformer, a transient effect may take place in the form of a momentary rush of current, due to the residual magnetism of the transformer iron. With the introduction of the new silicon steel for transformer cores, with the resulting increase in flux densities, this transient effect has been materially magnified, and may, in some cases, reach dangerous proportions.

It is the object of this bulletin to present some facts with regard to this phenomenon, obtained by means of commercial apparatus, and to show how to protect the system from injury due to this cause.

2. *Acknowledgments.*—Valuable assistance in the preparation of the oscillograms has been rendered by Messrs. C. E. Bennett and A. C. Hobble, of the Electrical Engineering Department.

3. *Theory.*

(*a*) *Inductance without Iron.*—If an alternating e. m. f. $e = E_{max} \sin \theta$, be impressed upon a circuit containing resistance and inductance without iron, in series, this impressed e. m. f. will be consumed by the counter e. m. f. of the inductance, and by the drop through the resistance, and this must be true at every instant, i. e., for every point of the impressed e. m. f. wave. We have, therefore,

$$e = E_{max} \sin \theta = L \frac{di}{dt} + Ri \quad \ldots\ldots\ldots\ldots\ldots (1)$$

where

θ is the phase angle of the impressed e. m. f. $= 2\pi$ ft.

Since

$$\frac{di}{dt} = \frac{di}{d\theta}\frac{d\theta}{dt} = \frac{di}{d\theta} 2\pi f$$

$$L\frac{di}{dt} = 2\pi f L \frac{di}{d\theta} = X_L \frac{di}{d\theta}, \text{ where } X_L = \text{inductive reactance}$$

and

$$E_{max} \sin \theta = X_L \frac{di}{d\theta} + Ri$$

$$E_{max} \sin \theta \, d\theta = X_L \, di + Ri \, d\theta$$

$$-E_{max} \, d(\cos \theta) = X_L \, di + Ri \, d\theta$$

$$di = -\frac{E_{max}}{X_L} d\,(\cos\theta) - \frac{R}{X_L}\, i d\theta \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

If the circuit be closed at that point of the e. m. f. wave where $e = E_{max}$, i. e., when $\theta = 90° = \frac{\pi}{2}$, and if the resistance drop be assumed negligible, (2) becomes

$$di = -\frac{E_{max}}{X_L} d\,(\cos\theta) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

If the circuit be closed at different points of the e. m. f. wave, the current will rise to different values, and these values can now readily be investigated by means of the last equation.

Suppose for instance, that we close the circuit at the 90° point of the e. m. f. wave, i. e., when the e. m. f. is a maximum.
Integrating (3) from $\pi/2$ to π

$$\int_{\frac{\pi}{2}}^{\pi} di = -\frac{E_{max}}{X_L} \int_{\frac{\pi}{2}}^{\pi} d\,(\cos\theta) = \frac{E_{max}}{X_L}$$

which is the maximum current reached, since integrating from $\frac{\pi}{2}$ to $\pi + a$, where a is a constant, less than 2π, results in a value less than

$$\frac{E_{max}}{X_L}$$

showing that the current decreases from this point.

Suppose, in the next case, that the circuit be closed at the 0° point of the e. m. f. wave, i. e., when $e = 0$ and $\theta = 0$.
Integrating (3) from 0 to $\frac{\pi}{2}$ gives

$$\int_{o}^{\frac{\pi}{2}} di = -\frac{E_{max}}{X_L} \int_{o}^{\frac{\pi}{2}} d\,(\cos\theta) = +\frac{E_{max}}{X_L}$$

the same as before.
Integrating from 0 to π, however,

$$\int_{o}^{\pi} di = -\frac{E_{max}}{X_L} \int_{o}^{\pi} d\,(\cos\theta) = +2\,\frac{E_{max}}{X_L}$$

i. e., the maximum current obtained in this case is twice that obtained when the circuit is closed at the 90° point of the e. m. f. wave.

In a similar way it can be shown that by closing the circuit at any other point of the e. m. f. wave, the maximum current reached will lie

between

$$\frac{E_{\max}}{X_L} \text{ and } 2\frac{E_{\max}}{X_L}$$

In general, the current assumes its normal value only when the circuit is closed at that point of the impressed e. m. f. wave, where the permanent value of the current is zero. In the above case, where there is negligible resistance, this is the 90° point of the wave. The effect of the resistance is to move this point towards the zero point.

(*b*) *Inductance with Iron.*—The above calculations assume a constant inductance, i. e., a straight line magnetization curve, obtained by using an inductance without magnetic material as core. If an iron core be employed, such as is the case with the ordinary induction coil or the transformer, the inductance is not constant. As the flux density increases, the inductance decreases, until the iron is perfectly saturated. After this point is reached, the inductance remains constant at a small value, depending only upon the flux passing between the coil and the core through the air or non-magnetic material.

Since the flux is not any longer proportional to the current, the counter e. m. f. due to the inductance must be written

$$A\frac{dB}{d\theta}$$

where B = flux density and A = constant,
instead of

$$L\frac{di}{dt} \text{ or } X_L\frac{di}{d\theta}$$

and equation (1) becomes

$$e = E_{\max} \sin\theta = A\,\frac{dB}{d\theta} + Ri \qquad (4)$$

$$-E_{\max}\,d\,(\cos\theta) = AdB + Rid\,\theta$$

$$dB = -\frac{E_{\max}}{A}\,d\,(\cos\theta) - \frac{R}{A}\,id\,\theta \qquad (5)$$

Under normal conditions, the resistance drop due to the magnetizing current of a transformer is negligible, and

$$dB = \frac{E_{\max}}{A}\,d\,(\cos\theta)$$

The normal maximum value of B is then obtained by integrating dB from $\frac{\pi}{2}$ to π.

$$\int_{\frac{\pi}{2}}^{\pi} dB = B_{\max} = -\frac{E_{\max}}{A}\left[\cos\theta\right]_{\frac{\pi}{2}}^{\pi} = \frac{E_{\max}}{A}$$

$$\therefore \frac{E_{max}}{A} = B_{max} \text{ and } A = \frac{E_{max}}{B_{max}} \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Substituting (6) in (5)

$$dB = -B_{max}\, d\,(\cos\theta) - \frac{E_{max}}{B_{max}} Rid\,\theta \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Since the relation between the magnetizing current and the resulting flux can not be expressed mathematically in any practical equation, the magnetizing current necessary to produce the required flux according to the above equation can be determined only analytically, as follows:

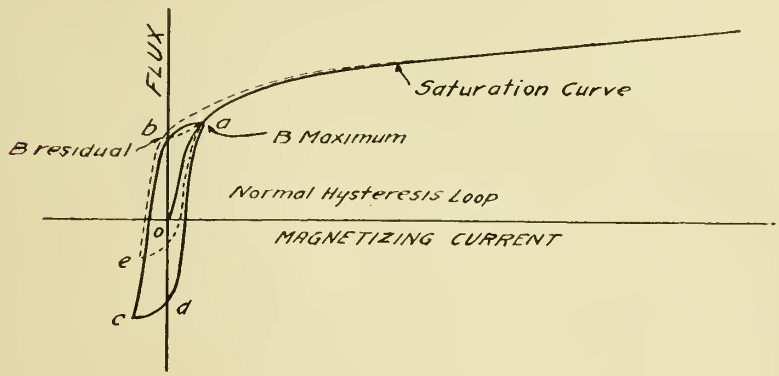


FIG. 1

Suppose Fig. 1 to represent the saturation curve of a transformer and the hysteresis loop for normal voltage and frequency. The hysteresis loop shows what residual magnetism remains in the iron after the current has been removed. *ob* and *od* represent this residual magnetism, depending upon whether the current has died down from a positive or a negative value.

Suppose the circuit is closed when the impressed e. m. f. passes through 0 from negative to positive, i. e., $\theta = 0$, and that the residual magnetism is *ob*. It is evident then that the change of flux to produce the counter e. m. f. must start from *b*. If equation (7) be rewritten in the form

$$\Delta B = -B_{max}\,\Delta\,(\cos\theta) - \frac{B_{max}}{E_{max}} Ri\,\Delta\,\theta \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

and small intervals of θ, say 10°, be taken, the actual conditions can very nearly be approached. Starting from *b*, the flux will follow a curve, such as the dashed curve between *b* and *a*, and will continue on the saturation curve. From equation (8), ΔB can be calculated for

each increment of 10°, starting from 0° in this case, and from Fig. 1 can be obtained the corresponding magnetizing current required to produce the total flux, $B_1 + \Delta B$, B_1 being the total flux at the beginning of the interval. After having determined the magnetizing current, the resistance drop effect is calculated, equal to

$$\frac{B_{\max}}{E_{\max}} Ri \Delta \theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

This will, however, reduce the value of ΔB, and a few trials will have to be made before the correct value of ΔB is found.

Proceeding in this manner, the flux and the corresponding magnetizing current may be determined for any number of cycles. For decreasing values of flux, the upper dashed curve in Fig. 1 has to be used. It will be found that the magnetizing current may reach formidable values under unfavorable conditions, particularly for the first cycle. The amplitude of the peaks decreases rapidly, the more so the larger the amplitude of the first peak, on account of the more pronounced effect of the resistance in that case.

4. *Method of Investigation.*—In Part II (*a*) will be taken up the actual measurements of the magnetizing current of a transformer upon closing the primary circuit at a predetermined point of the e. m. f. wave, and with a known residual magnetism in the iron. These measurements were made by means of oscillograms, showing the impressed e. m. f., the primary magnetizing current, and the secondary induced e. m. f.

Part II (*b*) takes up the calculations of the flux and magnetizing current for the conditions under which the oscillograms were taken. In order to do this, all the characteristics of the circuit and transformer having any bearing upon the magnetizing current were carefully obtained. The curves plotted show that there is very close agreement between the actual curves, as obtained by means of the oscillograms, and the calculated curves as obtained by means of the circuit and transformer characteristics. This agreement shows that it is possible to make calculations of these phenomena, that can be fully relied upon, and that it is unnecessary to resort to the oscillograph in order to obtain reliable results. It was therefore deemed sufficient for the investigation of the rest of the transformers, covered by this bulletin, to obtain the transformer data necessary to make the calculations as shown in Part III. These calculations cover the most critical condition only, namely, the rush of current upon closing the circuit at the 0° point of the e. m. f. wave with the residual magnetism in the same direction in which the increase of flux will take place upon closing the circuit.

In Part IV, is given the result of placing a resistance or air core inductance in series with the transformer, and it is shown how to calculate a resistance or inductance sufficient to limit the rush of current to safe values.

II. *(a)* Actual Measurements of Phenomena by Means of Oscillograms.

5. *Connections.*—Fig. 2 is a diagram of the connections used for obtaining the oscillograms. *G* is a 10 kw. 440-v. alternator, 60 cycles, with taps, so as to give either 3-phase or 2-phase current, as shown in Fig. 2*a*. Taps 1, 3 and 5 are used for 3-phase, taps 1-4, 2-6, for 2-phase. The closing switch was designed and built specially for the investigation of these phenomena.[1] It was attached to the end of the generator

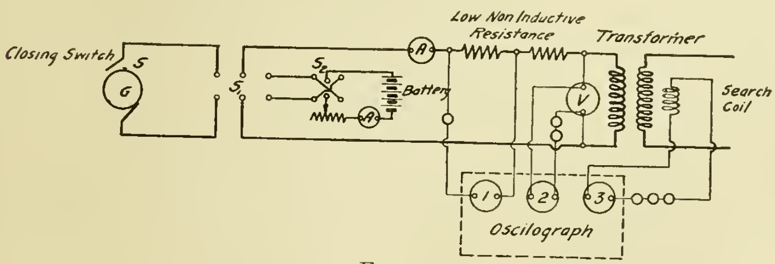


Fig. 2

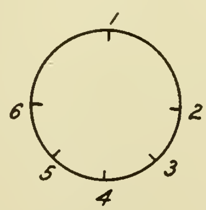


Fig. 2*a*

shaft, and can be set so as to close the circuit at any predetermined point of the e. m. f. wave. However, it would not operate satisfactorily at the normal speed of the generator, 1800 r. p. m. A speed of 650 r. p. m. was finally decided upon, which resulted in a frequency of 22 cycles.

6. *Transformer.*—A 5-kw. 60 cycles 2200, 1100/220, 110-volt transformer of the newest type was used in this test. It was connected for 110 volts primary, i. e., with the low tension coils in parallel. As the normal frequency is 60 cycles, and 22 cycles was used, the voltage

[1]By O. B. Wooten, Research Fellow, Engineering Experiment Station.

had to be reduced in proportion, i. e., the impressed voltage was $110 \times \frac{22}{60} = 40$ volts, to give normal magnetizing current.

As it was desirable to use as stiff a field as possible in the generator, in order to prevent too much of a voltage drop upon closing the transformer circuit, taps 2-3, (Fig. 2*a*), were used, giving 40 volts with about full field and 650 r. p. m. The oscillograms show that the voltage is kept up fairly well at the maximum rush of current.

7. *Residual Magnetism.*—The normal magnetizing current was obtained by impressing 110 volts at 60 cycles upon the transformer. The result is shown in the following table.

TABLE I

Volts	Current	Watts	Freq.
E	I_{ex}	W	F
110	.90	46.5	60

The maximum value of the exciting current $= .90 \times \sqrt{2} = 1.27$ amperes, and this is the current that produces the normal residual magnetism.

A series of experiments was made to ascertain the decrease of the residual magnetism after the removal of the e. m. f. These experiments are described in the Appendix. The following results were obtained:

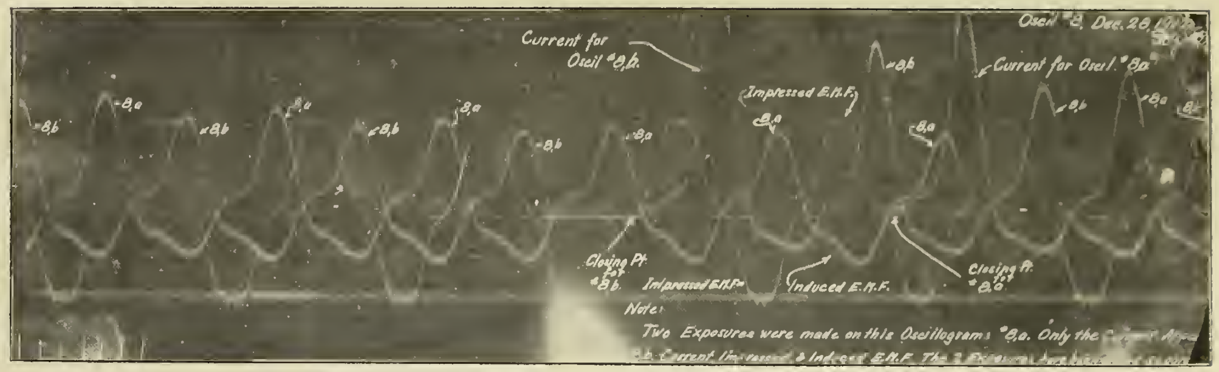
1. There is no decrease in the residual magnetism of transformers under normal conditions.

2. The decrease of residual magnetism due to vibration or shock is very small, almost negligible.

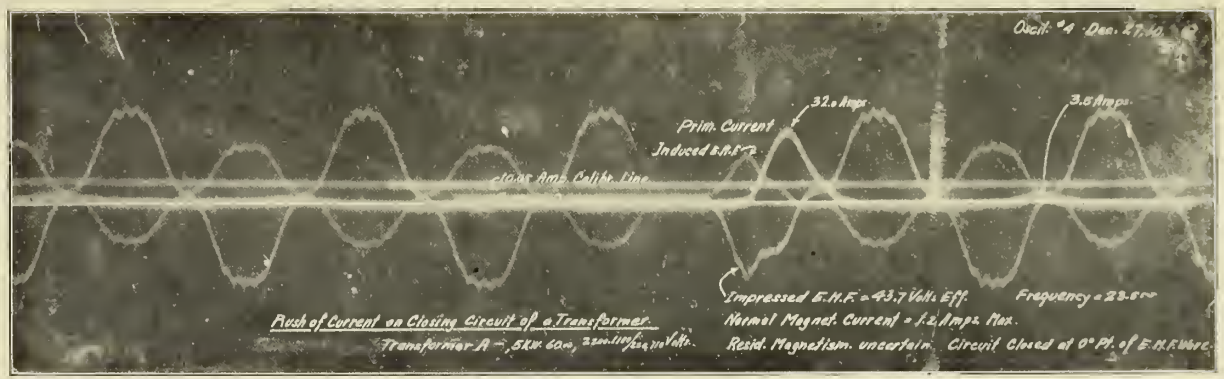
The oscillograms were taken with a residual magnetism in the iron that would remain after the removal of the normal voltage at normal frequency, which would be the case under normal operating conditions.

This residual magnetism was produced by means of direct current from a storage battery, as shown in Fig. 2. The current used was that corresponding to the normal exciting current of the transformer, the maximum value of which is 1.27 amp. Hence 1.27 amp. D. C. was used.

By means of a reversing switch, S_2, the current could be reversed, producing a residual magnetism in the opposite direction. In order to be sure that the correct residual magnetism was produced, the iron was sent through the regular hysteresis loop a number of times, at least



Oscillogram 8



Oscillogram 4

ten, by reversing the current by means of switch S_2. Let Fig. 3 represent the normal hysteresis loop. Suppose the residual magnetism, at the beginning, is at 2′. Sending $+I_m$ amperes through the transformer increases the magnetism to 1′ along the lower dashed curve. Opening the switch decreases the magnetism to 2′. Reversing the switch brings it near 3. Again opening the switch brings it near 4. Going through the same operations, the loop will approach 1-2-3-4 and, after a few reversals, practically coincide with it, so that when the switch is finally opened, the residual magnetism will be 0-2 or 0-4, according to whether the last current was $+I_m$ or $-I_m$.

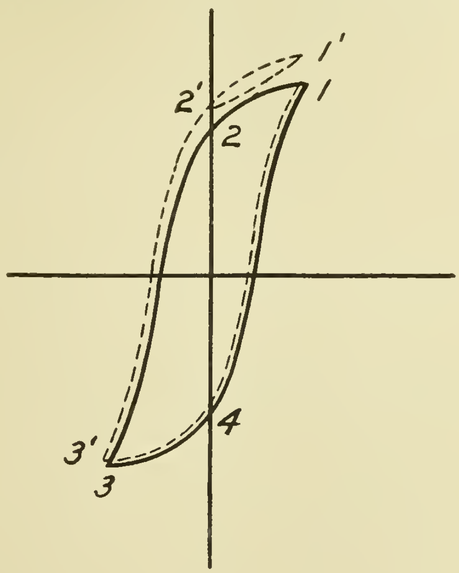


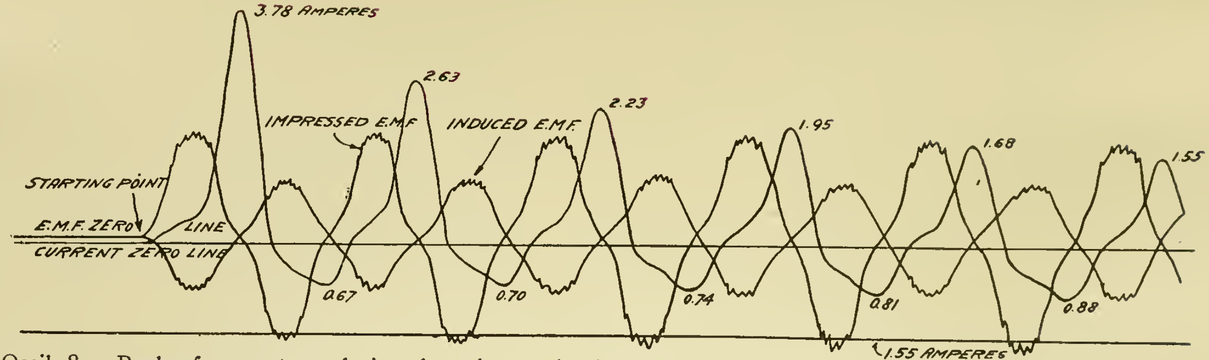
Fig 3

8. *The Oscillograms.*—Out of a total number of eleven oscillograms taken, four are here reproduced, as follows:

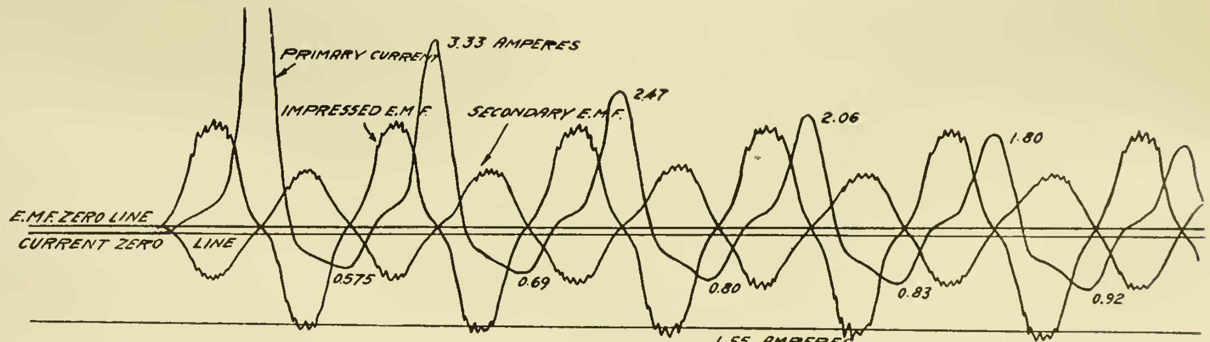
Oscil. 7. Circuit closed at 0° point of e. m. f. wave.
Residual magnetism, positive.
Maximum rush of current = 52.1 amp.

Oscil. 9. Circuit closed at 90° point (more accurately 85° point) of e. m. f. wave.
Residual magnetism, positive.
Maximum rush of current = 18.0 amp.

Oscil. 8 (*a*)* Circuit closed at 0° point of e. m. f. wave.
Residual magnetism, negative.
Maximum rush of current = 3.78 amp.



Oscil. 8*a*. Rush of current on closing the primary circuit of Transformer A. Circuit closed at 0° point of e. m. f. wave. Residual magnetism equals $-20 \times K$



Oscil 8*b*. Rush of current on closing the primary circuit of Transformer A. Circuit closed at 0° point of e. m. f. wave. Residual magnetism equals $-20 \times K$.

8 (*b*)* Same conditions.
Maximum rush of current, unknown.

Oscil. 4. Circuit closed at 0° point of e. m. f. wave.
Residual magnetism, uncertain.
Maximum rush of current = 32.0 amp.

By positive residual magnetism is meant that the magnetism was in the same direction in which the flux would increase upon closing the circuit.

II. (*b*) Theoretical Calculations from Transformer Data.

9. *Magnetization Curves and Hysteresis Loop.*—These were obtained in the following way. The transformer was connected as shown in Fig. 4. Direct current from a storage battery was supplied the high

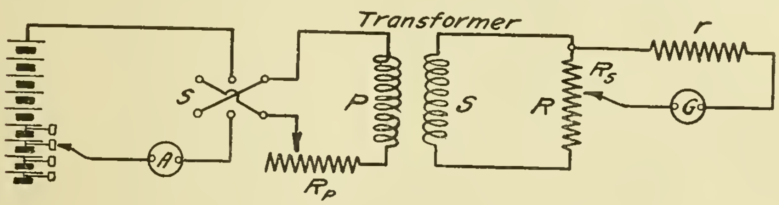


Fig. 4

tension side *P* of the transformer through a reversing switch, *S*, and a resistance R_p. The low tension side was connected to a high resistance *R*, and a D'Arsonval galvanometer, connected across a small part of the resistance, with a very high resistance in series. A change of flux in the transformer would then produce a deflection of the galvanometer coil, proportional to the total change of flux. *R*, R_s and *r* were not changed during the experiment, so that the deflections obtained, multiplied by a constant *K*, gave the flux density in the transformer core. In this investigation, the absolute flux density in gausses was not calculated, as it is only the relative flux values that are needed. The flux density is therefore, throughout this bulletin, expressed as a galvanometer deflection multiplied by a constant, *K*, K_1, K_2, etc. for different transformers.

To obtain the curves, the desired current was sent through the transformer primary, reversed a number of times to be sure that the iron had entered the corresponding loop, and the current left on in the

*Note.—Two exposures were made on Oscil. 8: 8 (*a*) in which the current only appeared; 8 (*b*) containing all quantities. The two exposures have been traced separately.

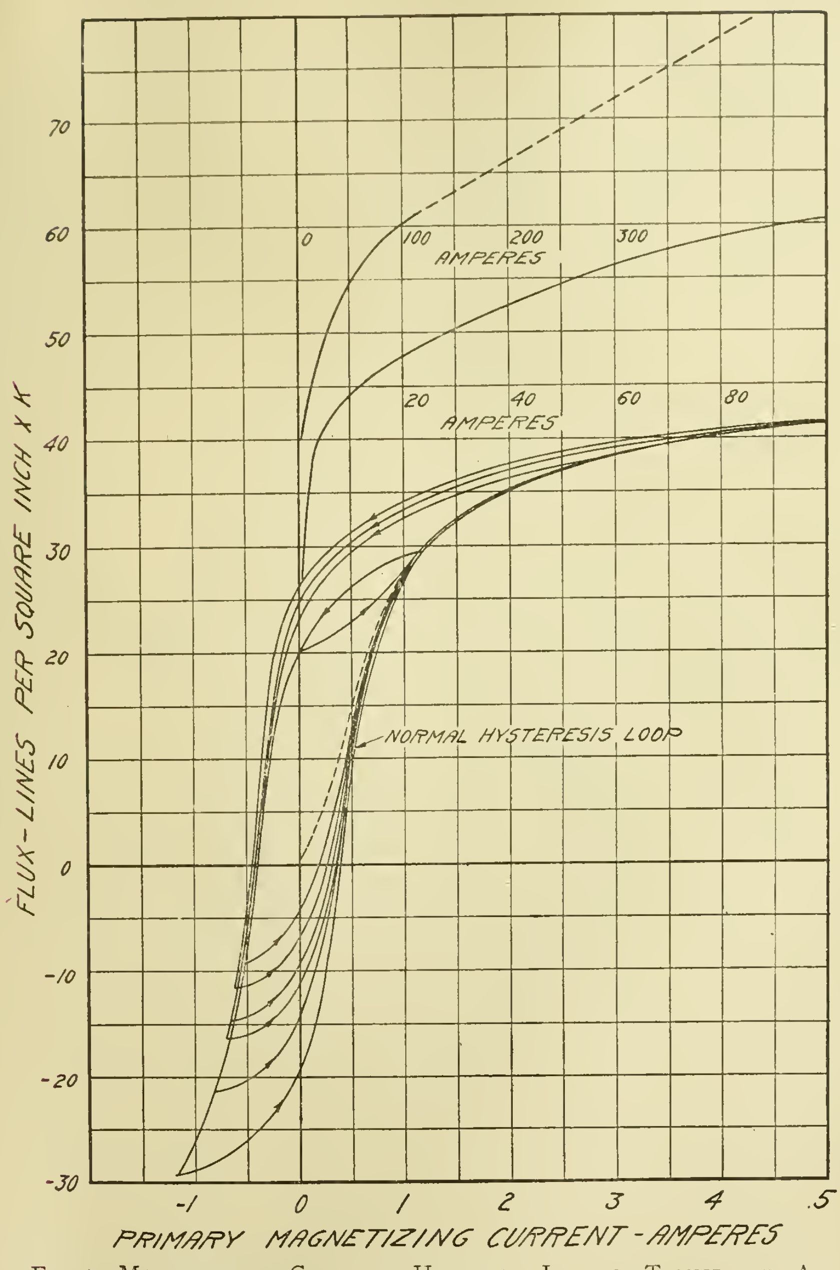


FIG. 5. MAGNETIZATION CURVE AND HYSTERESIS LOOP FOR TRANSFORMER A.

positive direction, corresponding to point 1 in Fig. 3. The galvanometer was then connected. Opening the circuit, the resulting deflection corresponded to a change of flux 1-2, reversing the current produced a change 2-3; opening it produced a change 3-4; again reversing it produced a change 4-1, completing the loop.

Fig. 5 shows the hysteresis loop and magnetization curves for the transformer used for the oscillograms. It was obtained by the method explained above. As an example, is given the galvanometer deflections for 1.22 amp.

TABLE 2

Current	Change	Deflection $= \Delta\phi \times K$	ϕ_{max}	Resid. Mag. ϕ
	1-2	9.5	$\frac{59.5}{2} = 29.75$	$29.75 - 9.5$
1.22	2-3	50.0		
	3-4	9.5	$\frac{59.5}{2} = 29.75$	$= 20.25$
	4-1	50.0		

The saturation curve was carried up to 119 amp. It is seen that at this point the curve has become a straight line, which means that the iron has become saturated, and the increase in flux is taking place only in the non-magnetic space between the iron and coil. Consequently, the curve can be extended indefinitely.

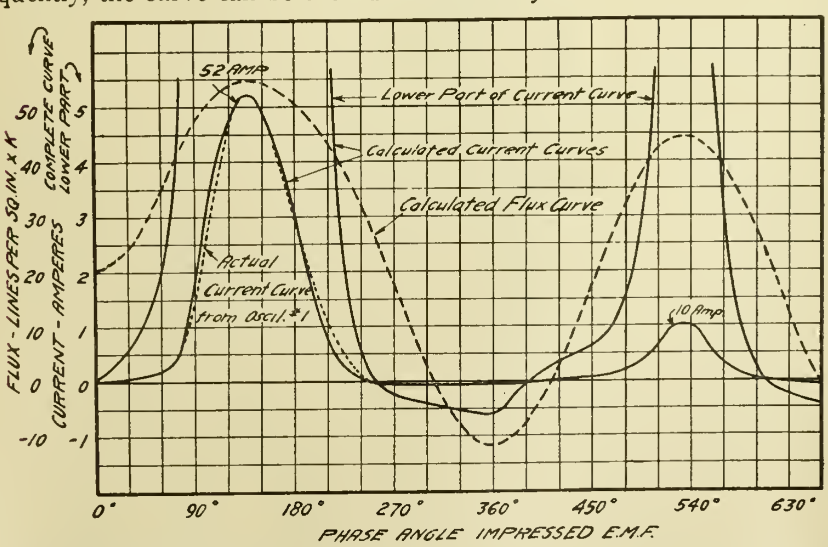


FIG. 6. RUSH OF CURRENT ON CLOSING THE PRIMARY CIRCUIT OF TRANSFORMER A. Circuit closed at 0° point of e. m. f. wave. Residual magnetism $= +20 \times K$.

9. *Calculations.*—The data needed for the calculations of the magnetizing current are as follows:

1. Normal hysteresis loop;
2. Magnetization curve up to straight line relation;
3. Total effective voltage impressed upon the transformer circuit;
4. Total resistance of circuit;
5. Total inductance of circuit.

Sine wave e. m. f. is assumed in these calculations. Equation (8), gives the relation:

$$\Delta B = -B_{\max} \Delta (\cos \theta) - \frac{B_{\max}}{E_{\max}} Ri \Delta \theta \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

assuming the circuit to have negligible inductance outside the transformer.

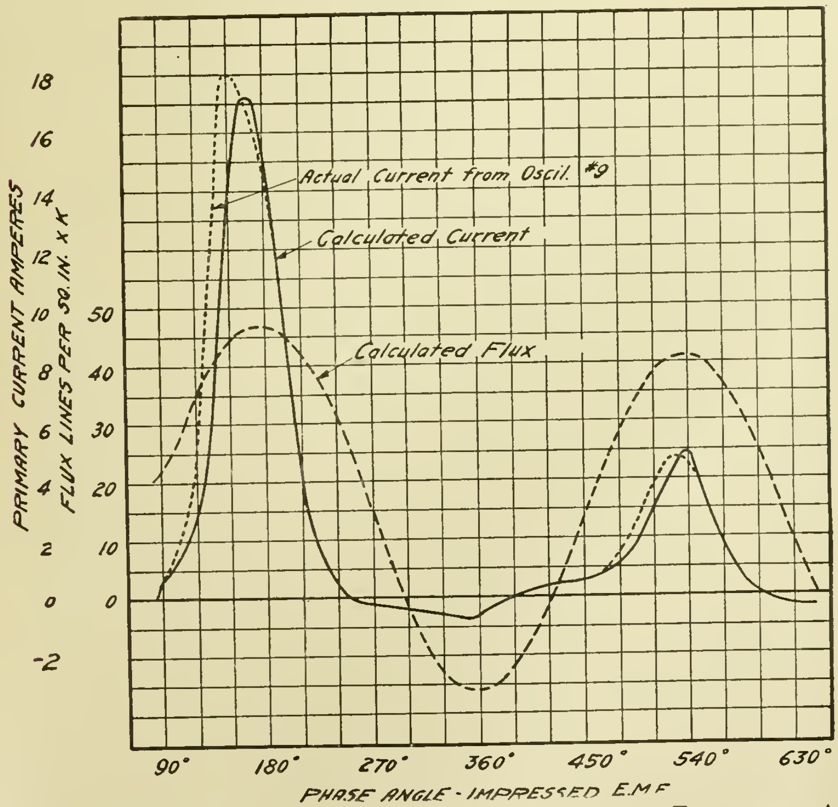


FIG. 7. RUSH OF CURRENT ON CLOSING THE PRIMARY CIRCUIT OF TRANSFORMER A. Circuit closed at 85° of e. m. f. wave. Residual magnetism equals $+20 \times K$.

$B =$ flux, $B_{max} =$ max. flux of normal hysteresis loop;

$E_{max} =$ max. impressed e. m. f. $= \sqrt{2}\, E_{eff}$;

$R =$ total resistance of circuit;

$i =$ instantaneous value of magnetizing current.

In the present case,

B_{max} (from Fig. 5) $= 29.5 \times K$, where $K =$ const.

$E_{eff} = 40$ volts. $E_{max} = \sqrt{2} \times 40 = 56.5$ volts.

$R = .745$ ohms.

Substituting in (8), for increments of θ of 10°, i. e. $\Delta\theta = 10° = .175$ radians,

$$\Delta B = -29.5\, K\, \Delta(\cos\theta) - .0685\, K\, i.$$

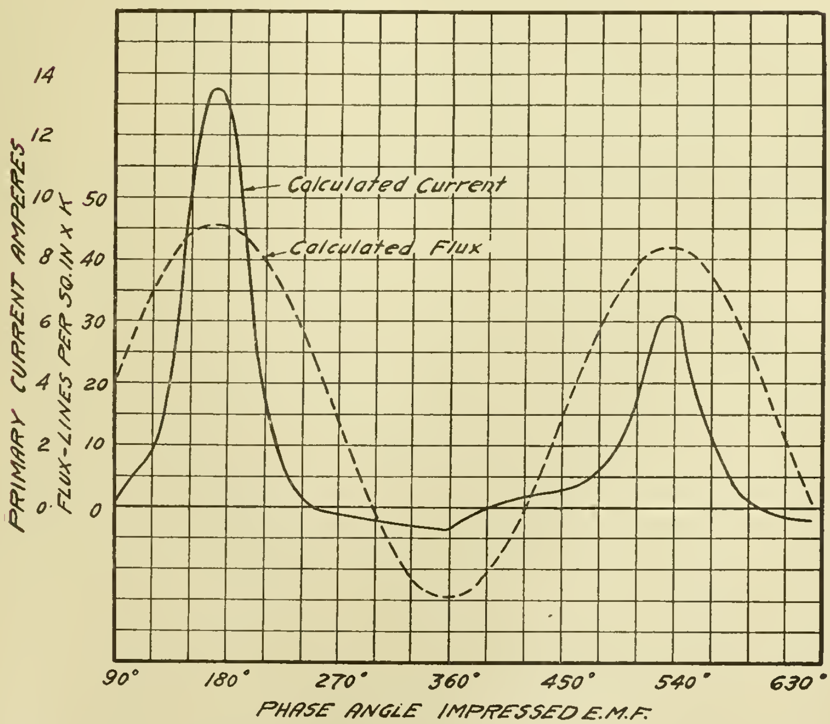


FIG. 8. RUSH OF CURRENT ON CLOSING THE PRIMARY CIRCUIT OF TRANSFORMER A. Circuit closed at 90° point of e. m. f. wave. Residual magnetism equals $+20 \times K$

In Table 3 are given the calculations for a number of different conditions, viz.,

CONDITIONS

	Closing Point on e. m. f. wave	Residual Magnetism	Impressed e. m. f. $_{eff}$	Frequency Cycles/Sec	Resist. of Circuit
Columns 4 to 7	0°	+ 20 K	40 volts	22	·745
Columns 8 to 11	85°	+ 20 K	40 volts	22	·745
Columns 12 to 15	90°	+ 20 K	40 volts	22	·745
Columns 16 to 19	0°	— 20 K	40 volts	22	·745
Columns 20 to 23	90°	— 20 K	40 volts	22	·745

These conditions correspond to those under which the oscillograms were taken.

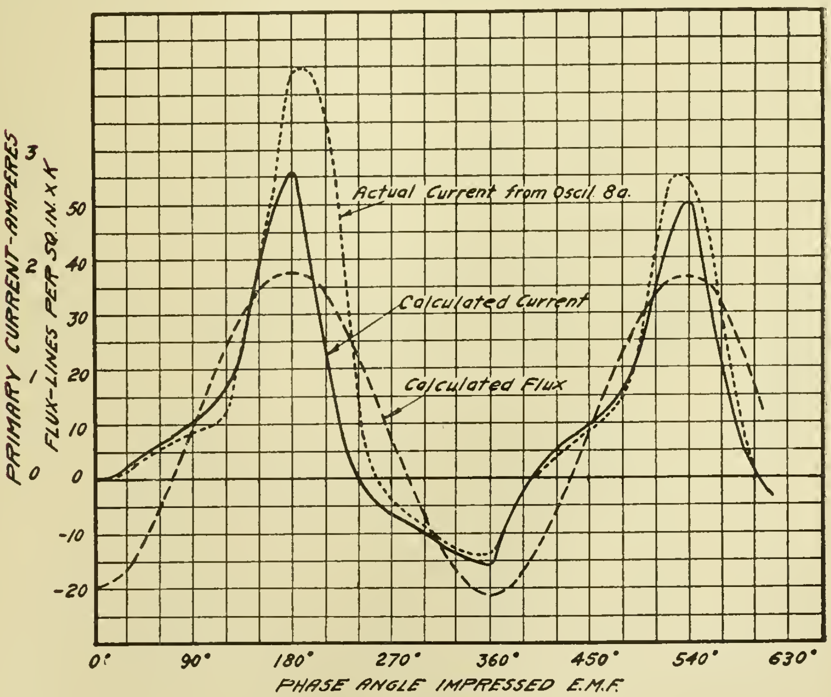


FIG. 9. RUSH OF CURRENT ON CLOSING THE PRIMARY CIRCUIT OF TRANSFORMER A. Circuit closed at 0° point of the e. m. f. wave. Residual magnetism equals $-20 \times K$.

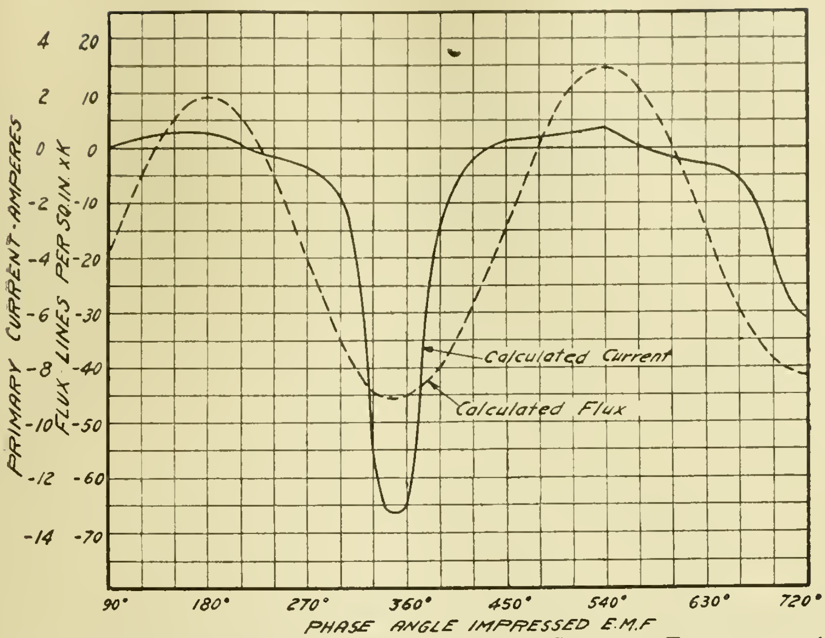


FIG. 10. RUSH OF CURRENT ON CLOSING THE PRIMARY CIRCUIT OF TRANSFORMER A. Circuit closed at 90° point of the e. m. f. wave. Residual magnetism = — 20 × K.

In Fig. 11 are plotted the calculated values of current, flux and impressed e. m. f. for the various conditions, to the same scale, in order to compare readily the effect of the closing point and the residual magnetism. Fig. 11 is a summary of Fig. 6 to 10 inclusive and Table 3.

11*a* corresponds to Table 3, Columns 4-7, and Fig. 6.
11*b* corresponds to Table 3, Columns 12-15, and Fig. 8.
11*c* corresponds to Table 3, Columns 16-19, and Fig. 9.
11*d* corresponds to Table 3, Columns 20-23, and Fig. 10.

11*e* represents the condition in which the circuit is closed at the 90° point of the e. m. f. wave with no residual magnetism. This is the condition for normal closing, since the flux and current then will enter at once upon their normal path. The same result would be obtained under conditions of Fig. 11*a* if the initial magnetism were negative maximum; under conditions in Fig. 11*b*, if the initial magnetism were 0; under conditions of Fig. 11*c*, if the initial magnetism were negative maximum, and under conditions of Fig. 11*d* if the initial magnetism were 0. It is seen that the closer the conditions come to these normal

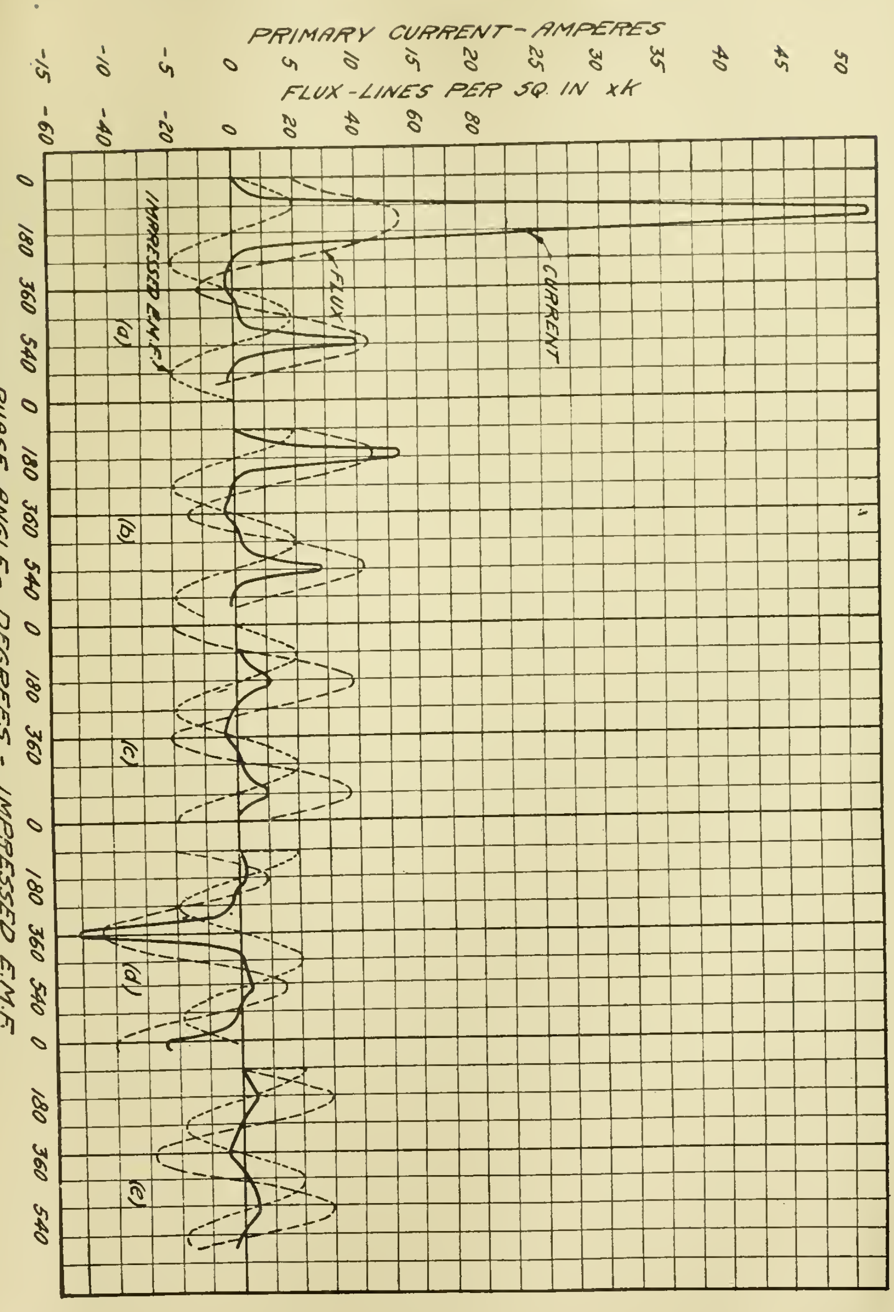


Fig. 11. Rush of Current on Closing the Primary Circuit of Transformer A.

TABLE 3—DETERMINATION OF FLUX AND CURRENT CURVES FOR TRANSFORMER A

IMPRESSED E. M. F. = 40 VOLTS EFFECTIVE AT 22 CYCLES RESISTANCE OF CIRCUIT = .745 OHMS.

			Circuit Closed at 0° Point of E. M. F. Wave Residual Magnetism = 20.0 K			
1	2	3	4	5	6	7
θ	$\cos\theta$	$-29.5\,K\,\Delta(\cos\theta)$	$\Delta B = -29.5\,K\,\Delta(\cos\theta) - .0685\,Ki_m$	Flux B	Mag. Current i_m	$.0685\,Ki_m$ b
0	+ 1.00	....	....	+ 20.0 K	+ .00	.00
10	.98	+ .59 K	+ .58 K	20.6	.20	.01
20	.94	1.18	1.15	21.8	.40	.03
30	.87	2.06	2.02	23.8	.60	.04
40	.77	2.95	2.88	26.7	.90	.06
50	.64	3.83	3.75	30.45	1.25	.085
60	.50	4.13	4.00	34.45	1.90	.13
70	.34	4.72	4.50	38.95	3.25	.22
80	+ .17	5.00	4.43	43.38	8.40	.575
90	.00	5.00	3.77	47.15	18.00	1.23
100	— .17	5.00	3.00	50.15	29.50	2.02
110	.34	5.00	2.30	52.45	39.50	2.70
120	.50	4.72	1.50	53.95	47.00	3.22
130	.64	4.13	.63	54.55	51.00	3.50
140	.77	3.83	+ .27	54.82	52.00	3.56
150	.87	2.95	— .45	54.37	49.50	3.40
160	.94	2.06	1.00	53.37	44.40	3.04
170	.98	1.18	1.37	52.00	37.60	2.57
180	1.00	+ .59	1.53	50.47	31.00	2.12
190	.98	— .59	2.11	48.36	22.20	1.52
200	.94	1.18	2.20	46.16	15.00	1.03
210	.87	2.06	2.65	43.51	8.60	0.59
220	.77	2.95	3.20	40.31	3.50	.25
230	.64	3.83	3.95	36.36	1.60	.12
240	.50	4.13	4.18	32.18	0.65	.045
250	.34	4.72	4.73	27.45	+ .10	+ .010
260	— .17	5.00	4.99	22.46	— .15	— .010
270	.00	5.00	4.98	17.48	.25	.020
280	+ .17	5.00	4.98	12.50	.32	.022
290	.34	5.00	4.97	7.53	.37	.025
300	.50	4.72	4.69	+ 2.84	.42	.029
310	.64	4.13	4.10	— 1.26	.45	.031
320	.77	3.83	3.80	5.06	.50	.034
330	.87	2.95	2.95	7.97	.53	.036
340	.94	2.06	2.02	9.99	.57	.039
350	.98	1.18	1.14	11.13	.60	.04
360	1.00	— .59	— .55	11.68	.60	.04
370	.98	+ .59	+ .62	11.06	.50	.03
380	.94	1.18	1.20	9.86	.25	.017
390	.87	2.06	2.06	7.80	— .07	— .0048
400	+ .77	+ 2.95	+ 2.94	— 4.86	+ .10	+ .007

TABLE 3—(Continued)—DETERMINATION OF FLUX AND CURRENT CURVES FOR TRANSFORMER A

IMPRESSED E. M. F. = 40 VOLTS EFFECTIVE AT 22 CYCLES RESISTANCE OF CIRCUIT = .745 OHMS.

	Circuit Closed at 85° Point of E. M. F. Wave Residual Magnetism = + 20.0 K				Circuit Closed at 90° Point of E. M. F. Wave Residual Magnetism = + 20.0 K			
1	8	9	10	11	12	13	14	15
θ	$\Delta B = -29.5K\,\Delta(\cos\theta) - .0685\,Ki_m$	Flux B	Mag. Current i_m	$.0685\,Ki_m$ b	$\Delta B = -29.5K\,\Delta(\cos\theta) - .0685\,Ki_m$	Flux B	Mag. Current i_m	$.0685\,Ki_m$ b
0								
10								
20								
30								
40								
50								
60								
70								
80	........*	+ 20.0 K	.00	.00				
90	+ 2.47	+ 22.47	.45	.031	....	20.0 K	.00	.00
100	4.93	27.40	.98	.067	+ 4.95 K	24.95	+ .74	.05 K
110	4.90	32.30	1.50	.100	4.92	29.87	1.17	.08
120	4.55	36.85	2.45	.168	4.60	34.47	1.82	.125
130	3.83	40.68	4.45	.305	3.92	38.39	3.00	.21
140	3.18	43.86	9.50	.650	3.43	41.82	6.00	.41
150	1.97	45.83	14.20	.98	2.30	44.12	10.00	.685
160	.90	46.73	17.00	1.165	1.18	45.30	12.75	.875
170	.015	46.75	17.10	1.17	+ .26	45.56	13.50	.92
180	— .48	46.26	15.60	1.07	— .30	45.29	13.00	.89
190	1.39	44.87	11.70	.80	1.27	44.02	10.00	.68
200	1.74	43.13	8.20	.56	1.64	42.38	6.70	.46
210	2.35	40.78	4.20	.29	2.30	40.08	3.55	.24
220	3.11	37.69	2.30	.16	3.08	37.00	2.00	.137
230	3.91	33.78	1.20	.08	3.90	33.10	1.00	.0685
240	4.16	29.62	+ .40	+ .03	4.15	28.95	+ .35	.024
250	4.72	22.90	— .06	.0	4.72	24.23	.00	.00
260	4.99	17.91	.20	— .01	4.99	19.24	— .15	.01
270	4.98	12.93	.26	.02	4.98	14.26	.25	.017
280	4.98	7.95	.33	.02	4.98	9.28	.30	.0206
290	4.97	+ 2.92	.38	.026	4.97	+ 4.31	.35	.024
300	4.69	— 1.77	.43	.03	4.69	— 0.38	.40	.027
310	4.10	5.87	.49	.034	4.10	4.48	.475	.0325
320	3.79	9.66	.55	.038	3.80	8.28	.525	.036
330	2.91	12.75	.60	.041	2.91	11.19	.575	.0394
340	2.02	14.77	.67	.046	2.02	13.21	.625	.043
350	1.13	15.90	.70	.048	— 1.13	14.34	.650	.045
360	— .54	16.44	.71	.048	— .54	14.88	.670	.046
370	+ .62	15.82	.40	.03	+ .62	14.25	.45	.035
380	1.20	14.62	.25	.02	1.20	13.06	.26	.018
390	2.06	12.54	— .06	— .00	2.06	11.00	— .08	.005
400	+ 2.95	9.59	+ .09	+ .006	+ 2.95	— 8.05	+ .08	.005

* 85° = θ

TABLE 3—(Concluded)—DETERMINATION OF FLUX AND CURRENT CURVES FOR TRANSFORMER A

IMPRESSED E. M. F. = 40 VOLTS EFFECTIVE AT 22 CYCLES RESISTANCE OF CIRCUIT = .745 OHMS.

	Circuit Closed at 0° Point of E. M. F. Wave Residual Magnetism = — 20.0 K				Circuit Closed at 90° Point of E. M. F. Wave Residual Magnetism = — 20.0 K			
1	16	17	18	19	20	21	22	23
θ	$\Delta B = -29.5K\,\Delta(\cos\theta) - .0685Ki_m$	Flux B	Mag. Current i_m	$.0685Ki_m$ b	$\Delta B = -29.5K\,\Delta(\cos\theta) - .0685Ki_m$	Flux B	Mag. Current i_m	$.0685Ki_m$ b
0	....	— 20.00 K	.00	.00				
10	+ 0.59	19.41	+ .01	.00				
20	1.18	18.23	.02	.00				
30	2.05	16.18	.12	+ .008				
40	2.94	13.24	.19	.01				
50	3.81	9.43	.26	.018				
60	4.11	5.32	.31	.021				
70	4.69	— 0.63	.38	.026				
80	4.97	+ 4.34	.45	.031				
90	4.97	9.31	.50	.034	....	— 20.0 K	.00	+ .00
100	4.96	14.27	.60	.041	+ 4.99	15.01	+ .15	.01
110	4.95	19.22	.72	.05	4.98	10.03	.25	.02
120	4.66	23.88	.88	.06	4.70	5.33	.30	.02
130	4.05	27.93	1.03	.08	4.10	— 1.23	.37	.03
140	3.73	31.66	1.42	.10	3.80	+ 2.57	.41	.03
150	2.82	34.48	1.90	.13	2.92	5.49	.45	.03
160	1.90	36.38	2.37	.16	2.03	7.52	.48	.03
170	1.00	37.38	2.65	.18	1.15	8.67	.50	.03
180	+ .40	37.78	2.80	.19	+ .56	9.23	.51	.03
190	— .75	37.03	2.30	.16	— .62	8.61	.40	.03
200	1.30	35.73	1.80	.12	1.20	7.41	.30	+ .02
210	2.15	33.58	1.25	.09	2.06	5.35	+ .05	.00
220	3.00	30.58	+ .65	.045	2.94	+ 2.41	— .10	— .01
230	3.85	26.73	+ .25	+ .02	3.82	— 1.41	.20	.01
240	4.13	22.60	.00	.00	4.11	5.52	.34	.02
250	4.71	17.89	— .15	— .01	4.69	10.21	.44	.03
260	4.98	12.91	.25	.02	4.96	15.17	.55	.04
270	4.98	7.93	.34	.02	4.95	20.12	.70	.05
280	4.97	+ 2.96	.37	.03	4.94	25.06	.90	.06
290	4.97	— 2.01	.43	.03	4.92	29.98	1.20	.08
300	4.68	6.69	.50	.035	4.60	34.58	1.90	.12
310	4.09	10.78	.56	.038	3.92	38.50	3.05	.21
320	3.79	14.57	.66	.044	3.42	41.92	6.00	.41
330	2.90	17.47	.69	.047	2.23	44.15	10.50	.72
340	2.01	19.48	.75	.051	1.16	45.31	13.00	.90
350	1.13	20.61	.78	.053	— .27	45.58	13.30	.91
360	— .53	21.14	.80	.055	+ .30	45.28	12.90	.89
370	+ .62	20.58	.50	.034	1.28	44.00	10.00	.69
380	1.20	19.38	.35	.024	1.53	42.47	5.10	.35
390	2.07	17.31	.15	— .01	2.30	40.17	3.50	.24
400	2.95	— 14.36	— .00	.00	+ 3.09	— 37.08	— 2.00	— .14

conditions, the less rush of current takes place. Fig. 11*c* comes very close to these normal conditions, while Fig. 11*a* is farthest away. Fig. 11*b* and *d* are practically identical, with the exception that in Fig. 11*b* the rush of current is positive, while in Fig. 11*d* it is negative.

10. *Agreement between Oscillograms and Calculated Curves.*—The values of flux and current from Table 3 have been plotted in Fig. 6 to 10 inclusive, together with the actual currents, as given by the oscillograms. The full lines give the calculated currents. The dashed lines give the calculated flux. The dotted lines give the actual currents.

From these plates it may be seen that the agreement between the actual curves and the calculated curves is very close. Indeed, for the first case, corresponding to Oscil. 7 and Table 3, Columns 4-7, the two current curves practically coincide. For the second case, corresponding to Oscil. 9 and Table 3, Columns 12-15, the maximum disagreement is only 4.5 per cent, while in the third case, corresponding to Oscil. 8*a* and Table 3, Columns 16-19, the disagreement is 25 per cent.

The closer agreement in the first case was to be expected, considering that a small variation in the residual magnetism in Oscil. 8 would have a greater effect than in Oscil. 7, on account of the dampening effect of the resistance in 7, while the resistance has practically no effect in 8. While the attempt was made to have the residual magnetism constant in all cases, it is possible that it may have varied a small amount. Assume, for instance, that the residual magnetism for Oscil. 8 was $-18.0\ K$ instead of $-20\ K$, the maximum positive flux would be approximately $37.8 + 2.0 = 39.8\ K$ corresponding to a current of 3.75 amp. (instead of 2.8 amp.) which is the current shown by the oscillogram.

However, the agreement between the oscillograms and the calculations is such as to warrant the conclusion that reliable results of the starting current of transformers can be obtained by calculations, if the complete data of the transformer and circuits are at hand, as tabulated on p. 15.

III. Calculation of Maximum Starting Current of Transformers of Various Types and Makes.

In the preceding section, have been given the starting currents of a 110-volt 60-cycles transformer by impressing upon it 40 volts at 22 cycles. While this resulted in normal magnetizing currents under normal operating conditions, the percentage of resistance drop in

terms of total impressed e. m. f. is much greater for the same current than if 110 volts were impressed.

For 110 volts, 60 cycles, equation (8) takes the following form: (the resistance of the circuit remaining .745 ohms)

$$\Delta B = -29.5\, K \Delta\, (\cos\theta) - \frac{29.5\ K}{110\sqrt{2}}\ Ri\, \Delta\, \theta$$

$$\Delta B = -29.5\, K \Delta\, (\cos\theta) - .025\ Ki$$

which shows that the effect of the resistance in decreasing ΔB, and consequently the current, is decreased by 110/40, or in proportion to the voltage.

In this section, calculations are given for the case in which the transformers are connected directly to constant potential busbars with sufficient power behind to maintain the voltage constant in spite of large starting currents. The potential in this case is the normal voltage of the transformers, and the resistance of the leads is assumed negligible.

The following transformers have been treated:

Designation	Capacity K.V.A.	VOLTS Primary	VOLTS Secondary	Freq.	Make	Year of Mfg.	Remarks
Transformer A	5	2200/1100	220/110	60	X	1910	Same transformer as was used in obtaining oscillograms.
Transformer B	5	2080/1040	460/230	60	X	Old Type	
Transformer C	50	2200/1100	440/220	60	Y	1910	
Transformer D	7½	440	110	60	Y	Old Type	
Transformer E	15	440/220	220/110	60	Z	1911	

The transformers will now be taken up in order, and the current calculated for the case where the circuit is closed at the 0° point of the e. m. f. wave, with the residual magnetism positive, i. e., for the conditions of Oscil. 7, which give the maximum rush of current.

Transformer A

5-kw., 2200, 1100/220, 110 volts, 60 cycles, new type, 110-volt winding used as primary.

Data

Hysteresis loop and magnetization curve are given in Fig. 5.

Normal effective voltage = 110 volts.

Resistance of circuit = resistance of transformer = .0253 ohms.

Maximum value of normal exciting current = 1.27 amp.

Hence equation (8) becomes

$$\Delta B = -29.5\ K\ \Delta\ (\cos\theta) - .00084\ Ki.$$

Table 4 gives the calculations for this and the following cases from 0 to 200°. For transformer A, it gives a maximum current of 390 amp., while the maximum value of the full load current is only $\sqrt{2} \times 45 =$ 64.3 amp., i. e., the maximum rush of current is 6.1 times normal full load.

Transformer B

5-kw. 2080, 1040/460, 230 V., 60 cycles, old type, 2080-volt winding used as primary.

Data

Hysteresis loop and magnetization curve are given in Fig. 12.

Normal eff. e. m. f. = 2080 volts.

Resistance of transf. (2080-volt winding) = 9.35 ohms.

Maximum value of normal exciting current = 0.1 amp.

From Fig. 12

$B_{max} = 26.25 \times K_4$

Normal residual magnetism = $20.0 \times K_4$.

$E_{max} = \sqrt{2} \times 2080 = 2940.$

Equation (8) becomes

$$\Delta B = -26.25 \times K_4 \Delta\ (\cos\theta) - .0146\ K_4 i.$$

From Table 4, Columns 8-11, the maximum current is 13.5 amp. or about 4 times the maximum value of the full load current, viz., $\sqrt{2} \times 2.4 = 3.4$ amp.

Transformer C

50 kw. 2200, 1100/440, 220 volts, 60 cycles, new type, 2200-volt side used as primary.

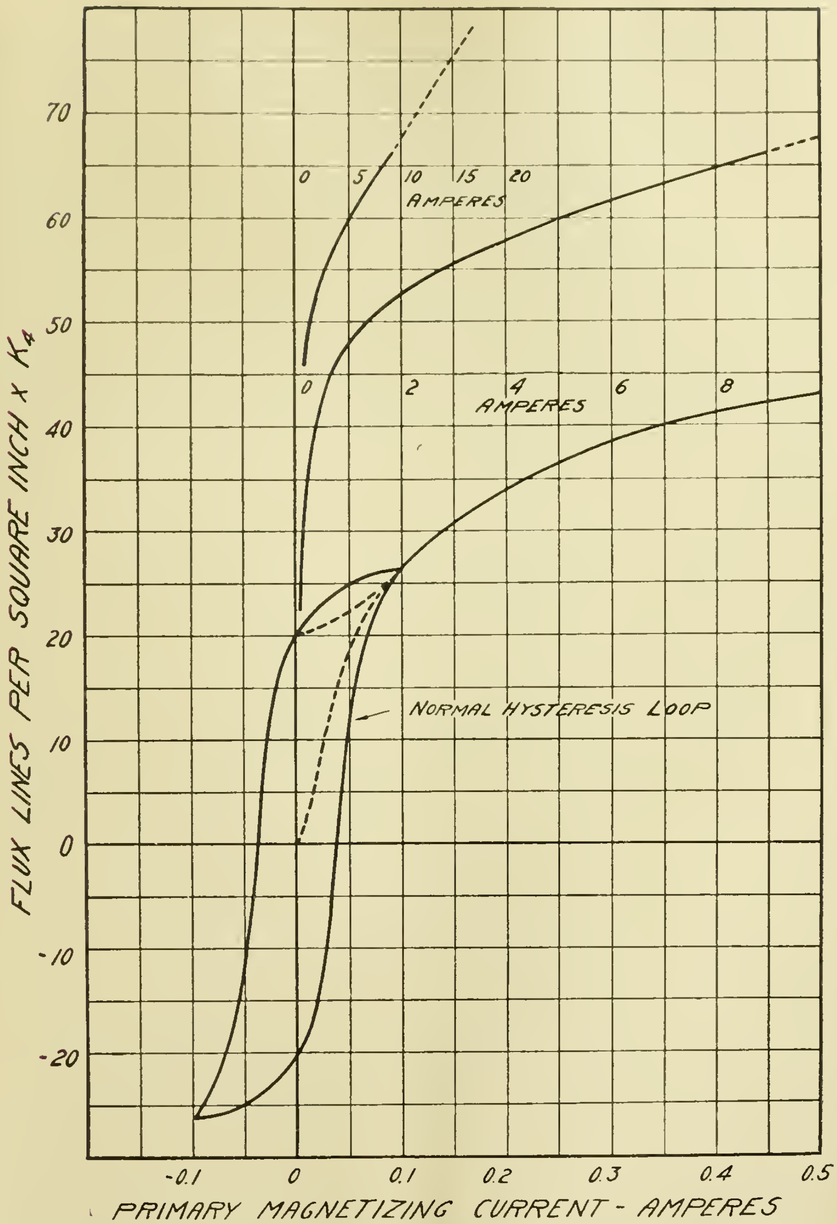


FIG 12. MAGNETIZATION CURVE AND HYSTERESIS LOOP OF TRANSFORMER B.

Data

Hysteresis loop and magnetization curve are given in Fig. 13.

Normal eff. e. m. f. = 2200 volts.

Resistance of transformer (2200-volt winding) = .446 ohms.

Maximum value of normal exciting current = .5 amp.

From Fig. 13

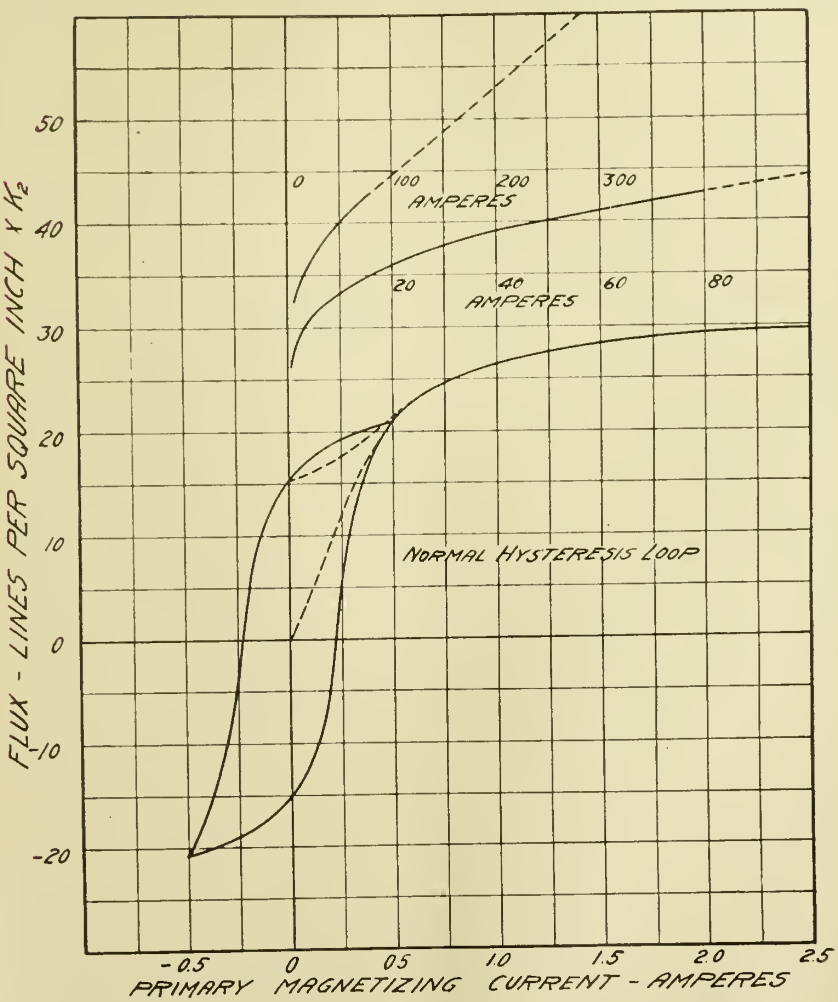


Fig. 13. Magnetization Curve and Hysteresis Loop for Transformer C.

$B_{max} = 20.75\ K_2$

Normal residual magnetism $= 15.25\ K_2$

$E_{max} = \sqrt{2} \times 2200 = 3110$ volts

Equation (8) becomes

$$\Delta B = -20.75\ K_2\ \Delta\ (\cos \theta) - .00052\ K_2\ i$$

From Table 4, Columns 12-15, the maximum current is 235 amp. or about 7.3 times the maximum value of the normal full load current, viz., $\sqrt{2} \times 22.7 = 32.1$ amp.

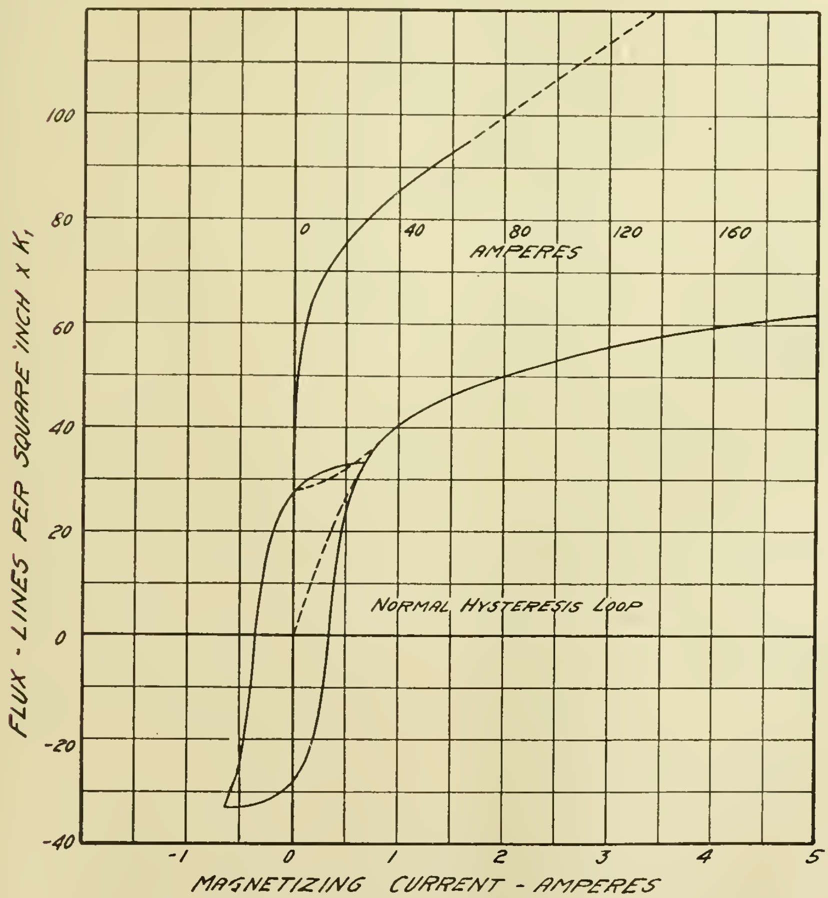


FIG. 14. MAGNETIZATION CURVE AND HYSTERESIS LOOP FOR TRANSFORMER D.

TABLE 4

DETERMINATION OF MAXIMUM RUSH OF CURRENT OF TRANSFORMERS

Negligible Resistance and Inductance in Primary Leads

		TRANSFORMER A 5 Kw. 110 Volts, Impressed at 60 Cycles Residual Magnetism = + 20.0 K				
		Circuit Closed at 0° Point of E. M. F. Wave Total Res. = .0253ω				
1	2	3	4	5	6	7
θ	$\cos\theta$	$-29.5K$ $\Delta\cos\theta$	$\Delta B =$ $-29.5K$ $\Delta\cos\theta$ $-.00084Ki_m$	Flux B	Mag. Current i_m	$.00084Ki_m$
0	+ 1.00	....	....	20.0 K	.00	.00
10	.98	+ .59 K	+ .6	20.6	.20	.00
20	.94	1.18	1.18	21.8	.40	.00
30	.87	2.06	2.06	23.9	.60	.00
40	.77	2.95	2.95	26.9	.90	.00
50	.64	3.83	3.83	30.7	1.30	.00
60	.50	4.13	4.13	34.8	1.90	.00
70	.34	4.72	4.72	39.5	3.60	.00
80	+ .17	5.00	5.00	44.5	11.00	.00
90	.00	5.00	4.98	49.5	27.00	+ .02
100	− .17	5.00	4.96	54.5	50.00	.04
110	.34	5.00	4.93	59.4	82.50	.07
120	.50	4.72	4.58	64.0	165.00	.14
130	.64	4.13	3.94	67.9	230.00	.19
140	.77	3.83	3.60	71.5	290.00	.24
150	.87	2.95	2.66	74.2	340.00	.29
160	.94	2.06	1.75	76.0	370.00	.31
170	.98	1.18	.86	76.9	385.00	.32
180	− 1.00	+ .59	+ .26	77.2	390.00	.33
190	.98	− .59	− .91	76.3	380.00	.32
200	− .94	− 1.18	− 1.47	74.8	345.00	.29

TABLE 4—(Continued)

DETERMINATION OF MAXIMUM RUSH OF CURRENT OF TRANSFORMERS

NEGLIGIBLE RESISTANCE AND INDUCTANCE IN PRIMARY LEADS

TRANSFORMER B 5 Kw. 2080 Volts Impressed at 60 Cycles Residual Magnetism = 20.0 K					TRANSFORMER C 50 Kw. 2200 Volts Impressed at 60 Cycles Residual Magnetism = + 15.25 K_2				
Circuit Closed at 0° Point of E. M. F. Wave Res. of Winding on 2080 Volt Side = 9.35ω					Circuit Closed at 0° Point of E. M. F. Wave Res. of Winding on 2200 Volt Side = .446ω				
8	9	10	11	12	13	14	15	16	17
$-26.25\ K_4$ $\Delta \cos\theta$	Δ B = $-26.25\ K_4$ $\Delta \cos\theta$ $-.0146\ K_4 i_m$	Flux B	Mag. Current i_m	$-.0146\ K_4 i_m$ b	$-20.75\ K_2$ $\Delta \cos\theta$	Δ B = $-20.75\ K_2$ $\Delta \cos\theta$ $-.00052$ $K_2 i_m$	Flux B	Mag. Current i_m	.00052 $K_2 i_m$
....	....	20.0 K_4	.00	.00	....	....	+ 15.25 K_2	.00	.00
+ .53 K_4	+ .53	20.5	.015	.00	+ .42 K_2	+ .42 K_2	15.70	.05	.00
1.06	1.06	21.6	.04	.00	.84	.84	16.5	.15	.00
1.84	1.84	23.4	.065	.00	1.45	1.45	18.00	.30	.00
2.63	2.63	26.0	.095	.00	2.08	2.08	20.00	.40	.00
3.42	3.42	29.4	.135	.00	2.70	2.70	22.70	.55	.00
3.68	3.68	33.1	.185	.00	2.90	2.90	25.60	.90	.00
4.22	4.22	37.3	.27	.00	3.32	3.32	28.90	1.70	.00
4.48	4.48	42.8	.50	.00	3.53	3.53	32.40	7.50	.00
4.48	4.48	47.3	.90	.00	3.53	3.52	35.90	20.00	.01 K_2
4.48	4.48	51.8	1.80	.00	3.53	3.51	39.40	41.00	.02
4.48	4.43	56.2	3.30	+ .05	3.53	3.49	42.90	80.00	.04
4.22	4.14	60.3	5.30	.08	3.32	3.26	46.20	120.00	.06
3.68	3.57	63.9	7.70	.11	2.90	2.82	49.00	150.00	.08
3.42	3.28	67.2	9.80	.14	2.70	2.60	51.60	185.00	.10
2.63	2.46	69.7	11.50	.17	2.08	1.97	53.60	205.00	.11
1.84	1.66	71.4	12.50	.18	1.45	1.33	54.90	225.00	.12
1.06	.85	72.3	13.25	.19	.84	.72	55.60	230.00	.12
+ .53	+ .33	72.6	13.50	.20	+ .42	+ .30	55.90	235.00	.12
− .53	− .72	71.9	12.90	.19	− .42	− .54	55.40	228.00	.12
− 1.06	− 1.24	70.7	12.00	.18	− .84	− .95	54.40	215.00	.11

TABLE 4—(Concluded)

DETERMINATION OF MAXIMUM RUSH OF CURRENT OF TRANSFORMERS

NEGLIGIBLE RESISTANCE AND INDUCTANCE IN PRIMARY LEADS

TRANSFORMER D 7.5 Kw. 440 Volts Impressed at 60 Cycles Residual Magnetism $= + 28\,K_1$					TRANSFORMER E 15 Kw. 440 Volts Impressed at 60 Cycles Res. Magnetism $= + 25.0\,K_3$				
Circuit Closed at 0° Point of E. M. F. Wave Res. of Winding on 440 Volt Side $= .26\omega$					Circuit Closed at 10° Point of E. M. F. Wave Res. of Winding on 440 Volt Side $= .195\omega$				
18	19	20	21	22	23	24	25	26	27
$-33\,K_1$ $\Delta\cos\theta$	$\Delta B =$ $-33\,K_1$ $\Delta\cos\theta$ $-.0024\,K i_m$	Flux B	Mag. Current i_m	$.0024\,K_1 i_m$ b	$-38\,K_3$ $\Delta\cos\theta$	$\Delta B =$ $-38\,K_3$ $\Delta\cos\theta$ $-.0021$ $K_3 i_m$	Flux B	Mag. Current i_m	$.0021\,K_3 i_m$ b
....	....	$28.0\,K_1$	.00	.00	....	....	$25.0\,K_3$	.. .00	.00
+ $.66\,K_1$	$.66\,K_1$	28.7	.20	.00	+ $.76\,K_3$	+ $.76\,K_3$	25.8	.05	.00
1.32	1.32	30.0	.30	.00	1.52	1.52	26.3	.10	.00
2.30	2.30	32.3	.50	.00	2.65	2.65	29.0	.30	.00
3.30	3.30	35.6	.75	.00	3.80	3.80	32.8	.50	.00
4.30	4.30	39.9	.97	.00	4.95	4.95	37.8	.75	.00
4.60	4.60	44.5	1.30	.00	5.30	5.30	43.1	1.20	.00
5.30	5.30	49.8	2.00	.00	6.10	6.10	49.2	2.00	.00
5.60	5.60	55.4	3.00	.00	6.50	6.50	55.7	3.50	.00
5.60	5.60	61.0	5.00	.00	6.50	6.50	62.2	10.00	.00
5.60	5.60	66.6	9.00	.00	6.50	6.50	68.7	20.00	.00
5.60	5.60	72.2	16.00	.00	6.50	6.40	75.1	42.00	+ .088
5.30	5.20	77.4	24.00	$.06\,K_1$	6.10	5.90	81.0	90.00	.19
4.60	4.50	81.9	32.00	.08	5.30	5.00	86.0	140.00	29
4.30	4.20	86.1	42.00	.10	4.95	4.60	90.6	190.00	.40
3.30	3.20	89.3	51.00	.12	3.80	3.30	93.9	220.00	.46
2.30	2.20	91.5	57.00	.14	2.65	2.10	96.0	240.00	.50
1.32	1.20	92.7	60.50	.14	+ 1.52	1.00	97.0	250.00	.53
+ .66	+ .5	93.2	62.00	.15	+ .76	+ .26	97.3	255.00	.54
— .66	— .8	92.4	60.50	+ .14	— .76	— 1.26	96.0	240.00	.50
— 1.32	.00	.00	.00	.00	— 1.52	— 2.00	94.0	220.00	+ .46

Transformer D

7½-kw. 440/110 volts, 60 cycles, old type, 440-volt side used as primary.

Data

Hysteresis loop and magnetization curve are given in Fig. 14.

Normal eff. e. m. f. = 440 volts.

Resistance of transformer (440 volt-winding) = .26 ohms.

Maximum value of normal exciting current = .655 amp.

From Fig. 14

$B_{max} = 33.0 \times K_1$

Normal residual magnetism = 28.0 K_1

$E_{max} = \sqrt{2} \times 440 = 622.5$ volts.

Equation (8) becomes

$$\Delta B = -33\, K_1\, \Delta\, (\cos\theta) - .0024\, K_1\, i$$

From Table 4, Columns 16-19, the maximum current is 62 amp. or about 2.6 times normal full load current, viz., $\sqrt{2} \times 17 = 24.1$ amp.

Transformer E

440, 220/220, 110 volts, 60 cycles, new type, 440-volt side used as primary.

Data

Hysteresis loop and magnetization curve are given in Fig. 15.

Normal eff. e. m. f. = 440 volts.

Resistance of transformer (440-volt winding) = .195 ohms.

Maximum value of normal exciting current = .87 amp.

From Fig. 15

$B_{max} = 38.0\, K_3$

Normal residual magnetism = 25.0 K_3

$E_{max} = \sqrt{2} \times 440 = 622.5$ volts

Equation (8) becomes

$$\Delta B = -38.0\,K_3\,\Delta\,(\cos\theta) - .0021\,K_3 i$$

From Table 4, Columns 20-23, the maximum current is 255 amp. or 5.3 times the maximum value of the normal full load current, viz., $\sqrt{2} \times 34.1 = 48.2$ amp.

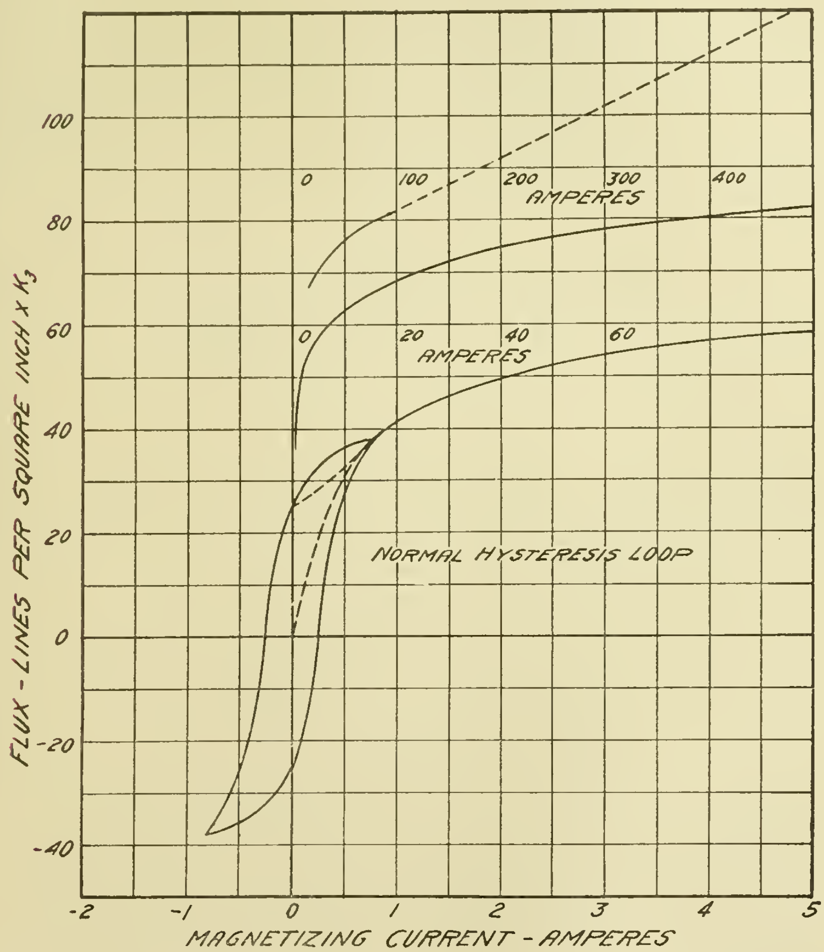


FIG. 15. MAGNETIZATION CURVE AND HYSTERESIS LOOP FOR TRANSFORMER E.

IV. Effect of Resistance and Inductance in Series with Transformer Primary.

In the preceding section, have been presented the results of connecting some transformers directly to the busbars with negligible resistance in the leads. It was shown that with the old type of transformers, the initial rush of current may amount to two to four times normal full load current, while with the new type, with silicon steel cores, the initial rush may exceed seven times full load current.

The only remedy for reducing these abnormal currents, where the transformer is to be connected to constant potential busbars, is the introduction of resistance or inductance in series with the primary winding, i. e., that side of the transformer which is to be connected to

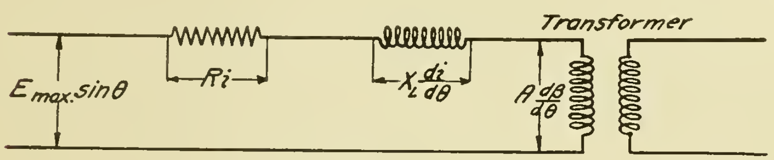


Fig. 16.

the power station. This inductance or resistance will take care of a part of the impressed e. m. f., leaving only a fraction of it to be taken care of by the counter e. m. f. of the transformer. This is shown diagrammatically in Fig. 16. The general equation then is

$$E_{\max} \sin\theta = A\frac{dB}{d\theta} + X_L \frac{di}{d\theta} + Ri \quad \ldots\ldots\ldots\ldots\ (10)$$

which, if solved for ΔB in the same way as in Part I, gives

$$\Delta B = -B_{\max}\,\Delta(\cos\theta) - \frac{B_{\max}}{E_{\max}} X_L\,(\Delta i) - \frac{B_{\max}}{E_{\max}} Ri\,(\Delta\theta) \quad \ldots\ldots\ (11)$$

This reduces to (8) if X_L is negligible.
R is the total resistance of the circuit, including the transformer primary, and X_L is the inductive reactance outside the transformer.

In the following, will be calculated the maximum rush of current with either resistance or inductance in series with the transformer primary for two of the silicon steel transformers, transformer A and transformer C.

Transformer A.

Case 1.—For negligible inductance outside the transformer equation (11) becomes

$$\Delta B = - B_{max} \Delta (\cos \theta) - \frac{B_{max}}{E_{max}} Ri (\Delta \theta)$$

Suppose now that $R = 1.21$[1] ohms, i. e., that the outside resistance is $1.21 - .025 = 1.185$ ohms, since the resistance of the transformer primary is .025 ohms. For 10° intervals of θ, $\Delta \theta = .175$.

$$B_{max} = 29.5 \text{ K}$$
$$E_{max} = 155 \text{ volts}$$
$$R = 1.21 \text{ ohms}$$
$$\Delta B = - 29.5 \, K \Delta (\cos \theta) - .0403 \, K i.$$

Case 2.—For negligible resistance in the leads, the total resistance of the circuit may be neglected, and (11) becomes

$$\Delta B = - B_{max} \Delta (\cos \theta) - \frac{B_{max}}{E_{max}} X_L \, (\Delta i)$$

Assume $X_L = 1.21$ ohms

$$\Delta B = - 29.5 \, K \Delta (\cos \theta) - .23 \, K \, (\Delta i)$$

Table 5 gives the calculations for these and the following cases. It is seen from this table that the maximum rush of current for Case 1 is 78.0 amp., and for the second case 86.0 amp. or less than twice full load current.

Transformer C.

Case 1—Negligible inductance outside transformer.

$$\text{For } R = \frac{\tfrac{1}{2} \text{ normal voltage}}{\text{full load current}} = \frac{1100}{24} = 43.5 \text{ ohms}$$

or $43.5 - .5 = 43$ ohms in series with the 2200-volt winding, equation (11) reduces to

$$\Delta B = - 20.75 \, K_2 \Delta (\cos \theta) - .051 \, K_2 \, i.$$

Case 2.—Negligible resistance outside transformer.

For $X_L = 43.5$ ohms in series with the 2200 volt-winding, equation (11) becomes, neglecting resistance:

$$\Delta B = - 20.75 \, K_2 \Delta (\cos \theta) - .29 \, K_2 \, (\Delta i).$$

From Table 5, Columns 12-14, it may be seen that the maximum rush of current for Case 1 is 50.0 amp., and for Case 2, 55 amp., i. e., in both cases less than twice full load current.

From the above calculations, it may be seen that the initial rush of current upon closing the primary circuit of a transformer can be limited to safe values by inserting either a resistance or an air core inductance in series with the primary circuit. In the particular cases above, the current was limited to less than twice full load current by

[1]This resistance multiplied by full load current, 45.5, gives a drop equal to half normal voltage: $1.21 \times 45.5 = 55$ volts.

TABLE 5

DETERMINATION OF MAXIMUM RUSH OF CURRENT OF TRANSFORMERS

INDUCTANCE OR RESISTANCE IN SERIES WITH TRANSFORMERS

TRANSFORMER A

5 KW. 2200, 1100/220, 110 VOLTS 60 CYCLES

RESIDUAL MAGNETISM = + 20.0 K

		Circuit Closed at 0° Point of E.M.F. Wave Total Resistance = 1.21ω Inductance = 0					Circuit Closed at 0° Point of E.M.F. Wave Total Resistance = 0. Ind. = 1.21ω			
1	2	3	4	5	6	7	8	9	10	11
θ	$\cos\theta$	$-29.5\,K_2$ $\Delta\cos\theta$	$\Delta B = -29.5\,K$ $\Delta\cos\theta$ $-.0403\,Ki_m$	Flux B	Mag. Current i_m	$.0403\,Ki_m$ b	$\Delta B = -29.5\,K$ $\Delta\cos\theta$ $-.23\,K\,\Delta\,i_m$	Flux B	Mag. Current i_m	$.23\,K$ $\Delta\,i_m$
0	+ 1.00	....	....	+ 20.0 K	.00	.00	.	+ 20.0 K	.00	.00
10	.98	+ .59 K	+ .60	20.6	.20	.00	+ .6 K	20.6	.00	.00
20	.94	1.18	1.20	21.8	.40	.00	1.20	21.8	.40	.00
30	.87	2.06	2.10	23.9	.60	.00	2.10	23.9	.60	.00
40	.77	2.95	2.95	26.9	.90	.00	2.90	26.8	.90	.00
50	.64	3.83	3.80	30.7	1.25	.05	3.70	30.5	1.25	+ .07 K
60	.50	4.13	4.00	34.7	1.90	.08	4.00	34.5	1.85	.08
70	.34	4.72	4.60	39.3	3.50	.14	4.40	38.9	3.25	.14
80	+ .17	5.00	4.60	43.9	10.00	.40	4.00	42.9	7.5	.32
90	.00	5.00	4.10	48.0	21.00	.85	3.30	46.2	15.0	.975
100	− .17	5.00	3.60	51.6	36.00	1.40	2.80	49.0	24.7	1.72
110	.34	5.00	3.00	54.6	50.00	2.00	2.50	51.5	35.5	2.20
120	.50	4.72	2.10	56.7	63.50	2.60	2.30	53.8	46.5	2.50
130	.64	4.13	1.30	58.0	72.00	2.90	1.78	55.58	56.75	2.50
140	.77	3.83	+ .70	58.7	78.00	3.10	1.55	57.13	66.5	2.35
150	.87	2.95	− .20	58.5	76.00	3.10	1.15	58.25	74.5	2.23
160	.94	2.06	− .70	57.8	70.00	2.80	.71	58.96	80.5	1.85
170	.98	1.18	.00	.00	.00	.00	.34	59.30	84.0	1.38
180	1.00	+ .59	.00	.00	.00	.00	+ .20	59.50	86.0	.80
190	.98	− .59	.00	.00	.00	.00	− .20	59.30	84.0	+ .46
200	.94	− 1.18	.00	.00	.00	.00	− .34	58.96	80.5	− .46
										− .80

TABLE 5—(Concluded)

DETERMINATION OF MAXIMUM RUSH OF CURRENT OF TRANSFORMERS

INDUCTANCE OR RESISTANCE IN SERIES WITH TRANSFORMERS

TRANSFORMER C

50 Kw. 2200, 1100/440, 220 VOLTS 60 CYCLES

RESIDUAL MAGNETISM = 15.25 K_2

Circuit Closed at 0° Point of E. M. F. Wave Total Resistance = 43.5ω. Ind. = 0					Circuit Closed at 0° Point of E. M. F. Wave Total Res. = 0. Ind. = 43.5ω			
12	13	14	15	16	17	18	19	20
$-20.75\,K_2\,\Delta\cos\theta$	$\Delta B = -20.75\,K_2\,\Delta\cos\theta - .051\,K_2 i_m$	Flux B	Mag. Current i_m	$.051\,K_2 i_m$	$\Delta B = -20.75\,K_2\,\Delta\cos\theta - 29\,K_2\,\Delta i_m$	Flux B	Mag. Current i_m	$.29\,K_2\,\Delta i_m$ b
....	....	15.25 K_2	.00	.00	....	15.25 K_2	.00	.00
+ .42 K_2	+ .42 K_2	15.70	.05	.00	+ .42 K_2	15.7	.05	.00
.84	.84	16.50	.15	.00	.84	16.5	.15	+ .03 K_2
1.45	1.45	18.00	.30	.00	1.45	18.0	.30	.045
2.08	2.08	20.00	.40	.00	2.08	20.0	.40	.03
2.70	2.70	22.70	.55	.00	2.70	22.7	.55	.045
2.90	2.90	25.60	.90	.00	2.80	25.5	.85	.09
3.32	3.20	28.80	1.65	.08 K_2	3.00	28.5	1.50	.19
3.53	3.17	32.00	7.00	.36	2.55	31.05	4.90	1.00
3.53	2.81	34.80	15.00	.72	2.20	33.25	9.50	1.34
3.53	2.25	37.05	25.00	1.28	1.70	34.95	15.75	1.80
3.53	1.70	38.75	36.00	1.85	1.55	36.5	22.50	1.96
3.32	1.00	39.75	45.00	2.30	1.35	37.85	29.25	1.96
2.90	.35	40.10	49.00	2.50	.90	38.75	36.00	1.96
2.70	+ .15	40.15	50.00	2.55	.80	39.55	42.50	1.89
2.08	— .3	39.85	45.50	2.32	.50	40.05	48.00	1.60
1.45	— .60	39.25	40.5	2.06	.35	40.40	51.70	1.07
.84	.00	.00	.00	.00	.15	40.55	54.00	.67
+ .42	.00	.00	.00	.00	+ .13	40.68	55.00	+ .29
— .42	.00	.00	.00	.00	— .13	40.55	54.0	— .29
— .84	.00	.00	.00	.00	— .15	40.40	51.7	— 1.07

inserting a resistance or inductive reactance equal to

$$R = X_L = \frac{\frac{1}{2}\text{ normal voltage}}{\text{full load current}}$$

in series with the primary.[1]

V. Summary and Conclusions

Table 6 gives the results of the calculations of the previous sections. Columns 9 and 10 give the results for the case in which the transformers are connected to the busbars with sufficient power behind to keep the voltage constant and with negligible resistance and inductance in the leads. Columns 11 and 12 give the results for the same conditions, but with a resistance of

$$R = \frac{\frac{1}{2} E}{I_{\text{full load}}}$$

in series with the primary.

Columns 13 and 14 give the results for the same conditions but with a reactance

$$X_L = 2\pi fL = \frac{\frac{1}{2} E}{I_{\text{full load}}}$$

in series with the primary.

Transformers A, C, and E are of recent manufacture with silicon steel cores, while B and D are of an old type.

In the preceding sections, it has been shown that transient currents amounting to several times the maximum value of full load current may occur upon closing the primary circuit of a transformer. While this transient current for the old type transformers may amount to two to four times full load current, it may rise above seven times the maximum value of full load current for transformers with cores made from silicon steels with high flux densities.

This transient current becomes a serious problem only for stations containing step-up transformers, that are connected directly to the station busbars through leads of negligible resistance and inductance. If the generators are belted and of only moderate capacity, the system may be flexible enough to stand the shock due to the enormous current that may follow upon closing the transformer switches. If, however, the generators are of large capacity and direct-connected, the shock may be sufficient to cause a rupture between the generator and the prime mover. It will readily be seen that this current will be of the same order as a direct short-circuit current of the generator.

[1]For calculations of inductance coils for this purpose reference is made to University of Illinois Bulletin 53, by Prof. Morgan Brooks and Mr. H. M. Turner, entitled, "Inductance of Coils".

TABLE 6

Summary of Results

1	2	3	4	5	6	7	8		9	10	11	12	13	14
		Volts							Transformer Connected Directly to Busbars		Connected to Busbars through Res. $= \frac{.5 E^1}{I_{\text{full load}}}$		Connected to Busbars through React. $= \frac{.5 E^1}{I_{\text{full load}}}$	
Transformers	Capacity K.V.A	Primary	Secondary	Freq. f	Make	Type	Winding Used	Eff. Value of Full Load Cur'nt Amp.	Max. Value of Max. Rush of Current amperes	Ratio to Max. Value of Full Load Current	Max. Value of Max. Rush of Current amperes	Ratio to Max. Value of Full Load Current	Max. Value of Max. Rush of Current amperes	Ratio to Max. Value of Full Load Current
A	5	2200 1100	220 110	60	X	New	110	45.50	390	6.1	78.0	1.21	86.0	1.34
B	5	2080 1040	460 230	60	X	Old	2018	2.40	13.5	4.0				
C	50	2200 1100	440 220	60	Y	New	2200	22.70	235	7.3	50.0	1.56	55.0	1.72
D	7.5	440	110	60	Y	Old	440	17.05	62	2.6				
E	15	440 220	220 110	60	Z	New	440	34.10	255	5.3				

[1]E = Normal Voltage of Primary Winding.

If each set of transformers be connected to one generator, the problem will consist simply in bringing the voltage up slowly with the transformers connected, but in cases where it becomes necessary to connect transformers to busbars of full potential, it becomes necessary, for safe operation, to insert in the primary circuit a resistance or inductance to limit the transient current to safe values. It has been shown that a resistance or inductive reactance amounting to

$$\frac{\frac{1}{2} \text{ normal voltage}}{\text{full load current}}$$

will limit the current to less than twice full load current under the most critical conditions.

This resistance or inductance needs to be in the circuit for only a very short time, since the current will fall down to below full load

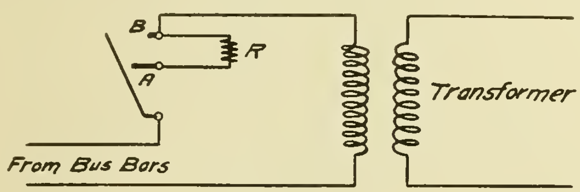


FIG. 17

current after a few cycles. The resistance or inductance may be connected as shown diagramatically in Fig. 17, where an extra contact, A, is provided on the switch, in such a way that, in closing the switch the contact A is reached before the main contact B is reached. It might be possible to provide such a contact even on oil switches. As the interval between the time the switch touches A until it touches B, need be only a fraction of one second, no change in the operating mechanism of the switch would be necessary. Usually, it takes an oil switch about 0.5 second to close, i. e., from the time it starts until the switch is closed. If the contact A is located 1/3 of the way from the closed position, it may take the contact 0.1 second to travel from A to B, and this time will be sufficient even for 25 cycle systems.

Appendix

Residual Magnetism.—It has been generally believed that residual magnetism is of a transient nature, i. e., that the magnetism that remains in the iron, after removing the magnetizing force, gradually decreases.

As this point is very important in connection with the starting current of transformers, an experiment was undertaken to ascertain whether the residual magnetism is a permanent quantity or not. Connections were made as shown in Fig. 18.

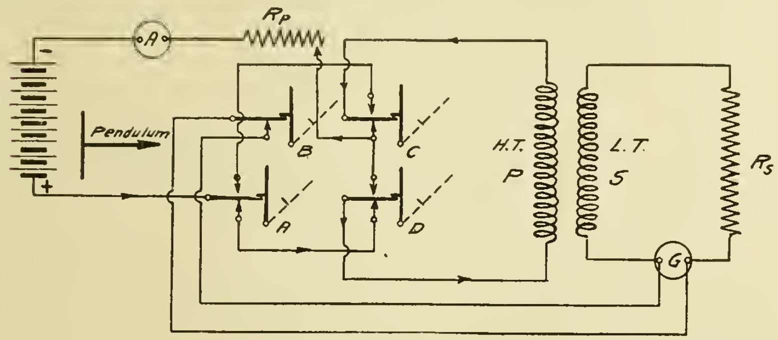


Fig. 18

Referring to the hysteresis loop of Fig. 3, it may be seen that, starting at 1, opening the circuit produces a change of flux corresponding to 1-2; reversing the current produces a change 2-3, etc. However, if the residual magnetism at 2 decreases before the current is reversed, the change of flux will be less than 2-3. Consequently, the problem consists in determining whether this change varies with the time elapsing between opening the circuit and closing it again in the opposite direction.

The time interval was controlled by means of a pendulum and four contacts, A, B, C and D. By tracing out the connections in Fig. 18, it will be seen that with all the levers in the upright position, the current flows through the circuit in the direction of the arrows, while the galvanometer G is shortcircuited by means of contact B. If now the pendulum is started from the position shown, lever A is first knocked down opening the primary circuit. The change of flux 1-2 does not produce any deflection of the galvanometer, because it is shortcircuited until lever B is knocked down. Finally, C and D are knocked down,

reversing the current, producing a change of flux 2-3, and since the galvanometer is no longer shortcircuited, this change produces a deflection of the galvanometer, proportional to the change of flux.

The shortest distance between A and D corresponded to about .1 second, and it could be increased to about ½ second.

The transformer used was the 5-kw. 2080, 1040/460, 230 volt, 60-cycle transformer, designated as transformer B. A resistance R_p, of 400 ohms, was inserted in the primary to decrease the time constant, and the current was maintained at the maximum value of the normal magnetizing current, namely, .1 amp.

1. The deflection corresponding to change 2-3 and .1 second interval between A and D was 48.0 cm. The interval was then in turn increased to ½ second, and by hand operation to 1 min., 5 min., 90 min., 12 hrs., and 24 hrs.

The deflection corresponding to the change 2-3 was in every case the same.

2. To ascertain whether there was any decrease of flux during the .1 second period, the resistance R_g amounting to more than 1 megohm was cut out. The sensitiveness of the galvanometer is such that 10^{-8} amp. corresponds to 1 mm. deflection. The transformer core has a cross-section of about 4 sq. in. and the number of turns of the 230-volt winding is 80. The normal flux density is about 50 000 lines per sq. in., and the resistance of the galvanometer circuit was less than 1000 ohms. Hence to produce a deflection of one cm. required 10^{-7} amp, or $10^{-7} \times 10^3 = 10^{-4}$ volts.

$$E = 10^{-4} = \frac{80 \times \phi}{10^8}$$

$$\phi = \frac{10^{-4}\ 10^8}{80} = \frac{10^4}{80} = 125 \text{ lines per sec.}$$

That is, it requires a change of flux of 125 lines per sec. to produce a large deflection of the galvanometer. As the total normal flux is 50 000 × 4 = 200 000 lines, this is less than 1/10 per cent, so that any material change of flux occurring within the first 1/10 sec. after opening A would be recorded. The time elapsing between A and B was less than .01 sec. The pendulum was stopped after knocking down B, so that any deflection occurring after the change 1-2 would be due to the decrease of the residual magnetism.

As a result of the several trials, not the slightest deflection could be detected.

3. It was finally attempted to determine the effect of vibration and blows upon the permanency of the residual magnetism. With the same connections as in 2, A and B were knocked down. The trans-

former core was then given a series of blows with a hammer. The first blow produced a deflection of about 50 cm., and the successive blows produced deflections decreasing very rapidly. This deflection corresponds to about $50/1000 = .05$ cm. with the resistance R_g in circuit.

Change 1-2 produced a deflection of	8.5 cm.
Change 2-3 produced a deflection of	48.0 cm.
Total change	= 56.5 cm.

This means that the maximum value of the normal flux corresponds to 28.3 centimeters deflection, and the residual magnetism to $28.3 - 8.5 = 19.8$ cm. deflection. Consequently, a deflection of .05 cm. corresponds to a decrease of residual magnetism due to severe blows of 5/20 per cent = 1/4 per cent.

The transformer was finally given continuous hard blows for 5 minutes (one blow every other second) after point 2 had been reached. With the resistance R_g cut out, the effect of the last blows could hardly be noticed. R_g was then replaced in the circuit and the deflection corresponding to the change 2-3 was observed. The result showed that the effect of the above severe treatment was to decrease the residual magnetism by 4 per cent.

Conclusion.—From the above, the conclusion seems justified that there is no decrease in the residual magnetism of a transformer under normal conditions, and that the decrease due to vibration and ordinary shocks is negligible.